## Course Title

## Principles of Photogrammetry

Chapter 3.5 .

## Pushbroom satellite images

If camera models is available，then the mathematics can be developed similar to the method described for aerial images
－Mathematics could be developed in a similar manner to aerial push－broom images
－Satellite perspective centres assumed to follow a certain
 ＇are）

Typical configuration of scan－ lines and perspective centres

测绘遥感信息工程国家重点实验室
－Similar concept to airborne pushbroom sensors，but different design to satisfy space requirements．
－Long focal length（eg 10 m ），narrow FOV（eg $<1^{\circ}$ ）
－Single sensor for vertical acquisition and stereo coverage
－Stereo coverage requires tilting satellite forward and backwards
－Agile－can point sideways as well as derive along－track for stereo
－Usually includes TDI correction for movement of satellite
－Products available from the supplier vary according to company policy and should be obtained from web sites：
－Original raw data
－Projected onto a plane at a certain level
－Images projected onto a rough DEM
－Accurate orthoimages
－Elevations will not be available．
－Typically used in many remote sensing software packages for geo－referencing remote sensing images
－Accuracies of the order of 0.5 to 1 pixel are usually achievable
－Accuracy will depend on magnitude of relief displacements in the images．May be large if：
－Angle of tilt of the satellite is large
－Images are high resolution
－Elevation differences are large in comparison to the resolution of the images．
测绘遥感信息工程国家重点实验室

Camera models often not available from satellite providers， hence geometry cannot be mathematically modelled
．Solution：
－Use of RPCs－Rational Polynomial Coefficients（also named RPFs）provided by satellite providers for use without GCPs

## General form of RPCs

$$
\begin{aligned}
& r_{n}=\frac{p 1\left(X_{n}, Y_{n}, Z_{n}\right)}{p 2\left(X_{n}, Y_{n}, Z_{n}\right)}=\frac{\sum_{i=0}^{m \mid} \sum_{j=0}^{m 2} \sum_{k=0}^{m 3} a_{i j k} X_{n}^{i} Y_{n}^{j} Z_{n}^{k}}{\sum_{i=0}^{n \mid} \sum_{j=0}^{n 2} \sum_{k=0}^{n 3} b_{n k} X_{n}^{j} Y_{n}^{j} Z_{n}^{k}}, \\
& c_{n}=\frac{p 3\left(X_{n}, Y_{n}, Z_{n}\right)}{p 4\left(X_{n}, Y_{n}, Z_{n}\right)}=\frac{\sum_{j=0}^{m 1} \sum_{j=0}^{m 2} \sum_{k=0}^{m 3} c_{i j k} X_{n}^{j} Y_{n}^{j} Z_{n}^{k}}{\sum_{i=0}^{n 1} \sum_{j=0}^{m 2} \sum_{k=0}^{n 3} d_{n k} X_{n}^{j} Y_{n}^{j} Z_{n}^{k}},
\end{aligned}
$$

$r_{n}$ and $c_{n}$ are normalised row and column pixel coordinates
Degree of polynomials limited to 3
$X_{n}, Y_{n}$, and $Z_{n}$ are normalised object coordinates $\mathrm{a}_{\mathrm{ijk}}, \mathrm{b}_{\mathrm{ijk}}, \mathrm{c}_{\mathrm{ijk}} \mathrm{d}_{\mathrm{ijk}}$ are polynomial coefficients called the rational function coefficients (RFC), also called RPCs which will ho of tho fnrm.

$$
\begin{aligned}
p=\sum_{j=0}^{m 1} \sum_{j=0}^{m 2} \sum_{k=0}^{m 3} a_{0 j} X^{\prime} Y^{\prime} Z^{k} & =a_{0}+a_{1} Z+a_{2} Y+a_{3} X+a_{4} Z Y \\
& +a_{5} Z X+a_{6} Y X+a_{7} Z^{2}+a_{8} Y^{2} \\
& +a_{9} X^{2}+a_{10} Z Y X+a_{11} Z^{2} Y \\
& +a_{12} Z^{2} X+a_{13} Y^{2} Z+a_{14} Y^{2} X \\
& +a_{15} Z X^{2}+a_{16} Y X^{2}+a_{17} Z^{3} \\
& +a_{14} Y^{3}+a_{10} X^{3}
\end{aligned}
$$

These RPCs usually only refer to the sensor and corrections need to be made to correct effects of elevations for images projected onto a plane at a certain level
－Inner orientation of image not required
－every pixel is at a fixed，calibrated position in the solid state image plane．
－determined by reference to a test range on the terrain
－corrections should not be necessary
－GNSS，star trackers and gyros used to determine camera tilts．
－Positions known to about 1 m ，tilts to 2 arc seconds
－In－track and cross－track position errors almost completely correlated with pitch and roll errors
－Only necessary to estimate pitch and roll because of high correlation between parameters for narrow FOV of sensor

Typical formulation for IKONOS II
Latitude, longitude and height of the object coordinates are $\varphi, \lambda$, h
Latitude, longitude and height offsets are LAT_OFF, LONG_OFF, HEIGHT_OFF, with LAT_SCALE, LONG_SCALE, HEIGHT_SCALE

$$
\begin{aligned}
P & =\frac{\phi-L A T_{-} O F F}{L A T_{-} S C A L E} \\
L & =\frac{\lambda-L O N G_{-} O F F}{L O N G_{-} S C A L E} \\
H & =\frac{h-H E I G H T_{-} O F F}{H E I G H T_{-} S C A L E}
\end{aligned}
$$

$$
Y=g(\phi, \lambda, h)=\frac{\operatorname{Num}_{L}(P, L, H)}{\operatorname{Den}_{L}(P, L, K H)}=\frac{c^{T} u}{d^{T} u}
$$

$\operatorname{Num}_{L}(P, L, H)=C_{1}+C_{2} L+C_{3} P C_{4} H+C_{5} L P+C_{6} L H+C_{7} P H+C_{8} L^{2}+$ $\mathrm{c}_{9} \mathrm{P} 2+\mathrm{C}_{10} \mathrm{H}^{2}+\mathrm{C}_{11} \mathrm{PLH} \mathrm{C}_{12} \mathrm{~L}^{3}+\mathrm{C}_{13} \mathrm{LP}^{2} \mathrm{C}_{14} \mathrm{LH}^{2}+\mathrm{C}_{15} \mathrm{~L}^{2} \mathrm{P}+\mathrm{C}_{16} \mathrm{P}^{3}+$ $\mathrm{C}_{17} \mathrm{PH}^{2}+\mathrm{C}_{18} \mathrm{~L}^{2} \mathrm{H}+\mathrm{C}_{19} \mathrm{P}^{2} \mathrm{H}+\mathrm{C}_{20} \mathrm{H}^{3}=\mathrm{c}^{\top} \mathrm{u}$
$\operatorname{Den}_{L}(P, L, H)=1+d_{2} L+d_{3} P+d_{4} H+d_{5} L P+d_{6} L H+d_{7} P H+d_{8} L^{2}+$ $\mathrm{d}_{9} \mathrm{P}^{2}+\mathrm{d}_{10} \mathrm{H}^{2}+\mathrm{d}_{11} P L H+\mathrm{d}_{12} \mathrm{~L}^{3}+\mathrm{d}_{13} \mathrm{LP}^{2}+\mathrm{d}_{14} \mathrm{LH}^{2}+\mathrm{d}_{15} \mathrm{~L}^{2} \mathrm{P}+\mathrm{d}_{16} \mathrm{P}^{3}+$ $\mathrm{d}_{17} \mathrm{PH}^{2}+\mathrm{d}_{18} \mathrm{~L}^{2} \mathrm{H}+\mathrm{d}_{19} \mathrm{P}^{2} \mathrm{H}+\mathrm{d}_{20} \mathrm{H}^{3}=\mathrm{d}^{\top} u$
Where
$u=\left[1 L P H L P L H P H L^{2} P^{2} H^{2} P L H L^{3} L P^{2} L H^{2} L^{2} P P^{3} P H^{2} L^{2} H P^{2} H\right.$ $\left.\mathrm{H}^{3}\right]^{\top}$
$\mathrm{c}=\left[\mathrm{c}_{1} \mathrm{c}_{2} \ldots \mathrm{c}_{20}\right]^{\top}$ and $\mathrm{d}=\left[\begin{array}{llll}1 & \mathrm{~d}_{2} & \ldots \mathrm{~d}_{20}\end{array}\right]^{\top}$

$$
X=h(\phi, \lambda, h)=\frac{\operatorname{Num}_{S}(P, L, H)}{{\operatorname{Den_{S}}}^{(P, L, K H)}=\frac{e^{T} u}{f^{T} u}, \frac{1}{}}
$$

$\operatorname{Num}_{L}(P, L, H)=e_{1}+e_{2} L+e_{3} P e_{4} H+e_{5} L P+e_{6} L H+e_{7} P H+$ $e_{8} L^{2}+e_{9} P 2+e_{10} H^{2}+e_{11} P L H e_{12} L^{3}+e_{13} L P^{2} e_{14} L H^{2}+e_{15} L^{2} P$ $+\mathrm{e}_{16} \mathrm{P}^{3}+\mathrm{e}_{17} \mathrm{PH}^{2}+\mathrm{e}_{18} \mathrm{~L}^{2} \mathrm{H}+\mathrm{e}_{19} \mathrm{P}^{2} \mathrm{H}+\mathrm{e}_{20} \mathrm{H}^{3}=\mathrm{e}^{\top} u$
$\operatorname{Den}_{L}(P, L, H)=1+f_{2} L+f_{3} P+f_{4} H+f_{5} L P+f_{6} L H+f_{7} P H+f_{8} L^{2}+$ $\mathrm{f}_{9} \mathrm{P}^{2}+\mathrm{f}_{10} \mathrm{H}^{2}+\mathrm{f}_{11} P L H+\mathrm{f}_{12} \mathrm{~L}^{3}+\mathrm{f}_{13} L \mathrm{P}^{2}+\mathrm{f}_{14} \mathrm{LH}^{2}+\mathrm{f}_{15} \mathrm{~L}^{2} \mathrm{P}+\mathrm{f}_{16} \mathrm{P}^{3}+$ $\mathrm{f}_{17} \mathrm{PH}^{2}+\mathrm{f}_{18} \mathrm{~L}^{2} \mathrm{H}+\mathrm{f}_{19} \mathrm{P}^{2} \mathrm{H}+\mathrm{f}_{20} \mathrm{H}^{3}=\mathrm{f}^{\top} \mathbf{u}$

Where
$e=\left[\begin{array}{llll}e_{1} & e_{2} & \ldots & e_{20}\end{array}\right]^{\top} \quad$ and $f=\left[\begin{array}{llll}1 & f_{2} & \ldots & f_{20}\end{array}\right]^{\top}$

For de-normalized image-space coordinates (Line, Sample)
Line $=$ image line number expressed in pixels with pixel zero as the centre of the first line
Sample $=$ pixel number with zero is the centre of the left-most sample Computed from:
(LINE_OFF, SAMP_OFF, LINE_SCALE, SAMP_SCALE),
Line $=Y$. LINE_SCALE + LINE OFF
Sample $=X$. SAMP_SCALE + SAMP_OFF
RPCs are subject to biases that limit the accuracy of geometric extraction to the order of 25 m

Determination of LPCs from 3 dimensional grid of points for IKONOS II

## IMAGE SPACE


－Errors in RPC developed by the satellite provider results in systematic errors in geometry in the image
－Apply a correction to positional parameters of exterior orientation， but in reality will compensate for sensor attitude errors
－Biases amount to about 20 pixels in $x$ and $y$ directions．
－Can be applied to the RPCs to correct for the bias．
－Accuracy after correction 0.6 m along－track， 0.3 m across－track and 0.8 m in heights

