

To carry out the linearization the collinearity equations are rewritten as follows and generalize it for object point j and camera i :

$$x_{ij} - x_0 + \Delta x_{ij} = -f \left(\frac{m_{11} (X_j - X_i^C) + m_{12} (Y_j - Y_i^C) + m_{13} (Z_j - Z_i^C)}{m_{31} (X_j - X_i^C) + m_{32} (Y_j - Y_i^C) + m_{33} (Z_j - Z_i^C)} \right) = Fx$$

$$y_{ij} - y_0 + \Delta y_{ij} = -f \left(\frac{m_{21} (X_j - X_i^C) + m_{22} (Y_j - Y_i^C) + m_{23} (Z_j - Z_i^C)}{m_{31} (X_j - X_i^C) + m_{32} (Y_j - Y_i^C) + m_{33} (Z_j - Z_i^C)} \right) = Fy$$

The linearization of the collinearity equations is achieved by Taylor's Series expansion, neglecting terms of higher order than 1. This involves partial differentiation of the collinearity equations. The equations are evaluated based on the substitution of the approximate values of the parameters.

$$\left(\frac{\partial F_x}{\partial X_i^c}\right) \Delta \hat{X}_i^c + \left(\frac{\partial F_x}{\partial Y_i^c}\right) \Delta \hat{Y}_i^c + \left(\frac{\partial F_x}{\partial Z_i^c}\right) \Delta \hat{Z}_i^c + \left(\frac{\partial F_x}{\partial \omega_i}\right) \Delta \hat{\omega}_i + \left(\frac{\partial F_x}{\partial \phi_i}\right) \Delta \hat{\phi}_i +$$

$$\left(\frac{\partial F_x}{\partial \kappa_i}\right) \Delta \hat{\kappa}_i + \left(\frac{\partial F_x}{\partial X_j}\right) \Delta \hat{X}_j + \left(\frac{\partial F_x}{\partial Y_j}\right) \Delta \hat{Y}_j + \left(\frac{\partial F_x}{\partial Z_j}\right) \Delta \hat{Z}_j + F_x^o - (x_{ij} - x_o + \Delta x_{ij}) = v_{x_{ij}}$$

$$\left(\frac{\partial F_y}{\partial X_i^c}\right) \Delta \hat{X}_i^c + \left(\frac{\partial F_y}{\partial Y_i^c}\right) \Delta \hat{Y}_i^c + \left(\frac{\partial F_y}{\partial Z_i^c}\right) \Delta \hat{Z}_i^c + \left(\frac{\partial F_y}{\partial \omega_i}\right) \Delta \hat{\omega}_i + \left(\frac{\partial F_y}{\partial \phi_i}\right) \Delta \hat{\phi}_i +$$

$$\left(\frac{\partial F_y}{\partial \kappa_i}\right) \Delta \hat{\kappa}_i + \left(\frac{\partial F_y}{\partial X_j}\right) \Delta \hat{X}_j + \left(\frac{\partial F_y}{\partial Y_j}\right) \Delta \hat{Y}_j + \left(\frac{\partial F_y}{\partial Z_j}\right) \Delta \hat{Z}_j + F_y^o - (y_{ij} - y_o + \Delta y_{ij}) = v_{y_{ij}}$$

Substituting terms for the partial derivatives

$$a_{11} \Delta \hat{X}_i^c + a_{12} \Delta \hat{Y}_i^c + a_{13} \Delta \hat{Z}_i^c + a_{14} \Delta \hat{\omega}_i + a_{15} \Delta \hat{\phi}_i +$$

$$a_{16} \Delta \hat{\kappa}_i + a_{17} \Delta \hat{X}_j + a_{18} \Delta \hat{Y}_j + a_{19} \Delta \hat{Z}_j + F_x^o - (x_{ij} - x_o + \Delta x_{ij}) = v_{x_{ij}}$$

$$a_{21} \Delta \hat{X}_i^c + a_{22} \Delta \hat{Y}_i^c + a_{23} \Delta \hat{Z}_i^c + a_{24} \Delta \hat{\omega}_i + a_{25} \Delta \hat{\phi}_i +$$

$$a_{26} \Delta \hat{\kappa}_i + a_{27} \Delta \hat{X}_j + a_{28} \Delta \hat{Y}_j + a_{29} \Delta \hat{Z}_j + F_y^o - (y_{ij} - y_o + \Delta y_{ij}) = v_{y_{ij}}$$

Let the collinearity equations be written as :

$$F_x = -f \frac{U}{W}$$

$$F_y = -f \frac{V}{W}$$

The elements of the linearized equations are therefore:

$$a_{11} = \frac{-f}{W} \left(-m_{11} + \frac{U}{W} m_{31} \right)$$

$$a_{21} = \frac{-f}{W} \left(-m_{21} + \frac{V}{W} m_{31} \right)$$

$$a_{12} = \frac{-f}{W} \left(-m_{12} + \frac{U}{W} m_{32} \right)$$

$$a_{22} = \frac{-f}{W} \left(-m_{22} + \frac{V}{W} m_{32} \right)$$

$$a_{13} = \frac{-f}{W} \left(-m_{13} + \frac{U}{W} m_{33} \right)$$

$$a_{23} = \frac{-f}{W} \left(-m_{23} + \frac{V}{W} m_{33} \right)$$

$$a_{14} = \frac{-f}{W} \left[-m_{13} (Y_j - Y_i^C) + m_{12} (Z_j - Z_i^C) - \frac{U}{W} \left\{ -m_{33} (Y_j - Y_i^C) + m_{32} (Z_j - Z_i^C) \right\} \right]$$

$$a_{24} = \frac{-f}{W} \left[-m_{23} (Y_j - Y_i^C) + m_{22} (Z_j - Z_i^C) - \frac{V}{W} \left\{ -m_{33} (Y_j - Y_i^C) + m_{32} (Z_j - Z_i^C) \right\} \right]$$

$$a_{15} = \frac{-f}{W} \left\{ \left[l_{11} (X_j - X_i^C) + l_{12} (Y_j - Y_i^C) + l_{13} (Z_j - Z_i^C) \right] - \frac{U}{W} \left[l_{31} (X_j - X_i^C) + l_{32} (Y_j - Y_i^C) + l_{33} (Z_j - Z_i^C) \right] \right\}$$

$$a_{25} = \frac{-f}{W} \left\{ \left[l_{21} (X_j - X_i^C) + l_{22} (Y_j - Y_i^C) + l_{23} (Z_j - Z_i^C) \right] - \frac{V}{W} \left[l_{31} (X_j - X_i^C) + l_{32} (Y_j - Y_i^C) + l_{33} (Z_j - Z_i^C) \right] \right\}$$

$$\begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix} = \begin{bmatrix} -\sin \omega m_{12} & \sin \omega m_{11} & -\cos \omega m_{11} \\ +\cos \omega m_{13} & & \\ -\sin \omega m_{22} & \sin \omega m_{21} & -\cos \omega m_{21} \\ +\cos \omega m_{23} & & \\ -\sin \omega m_{32} & \sin \omega m_{31} & -\cos \omega m_{31} \\ +\cos \omega m_{33} & & \end{bmatrix}$$

$$a_{16} = \frac{-f}{W} \left\{ m_{21} (X_j - X_i^C) + m_{22} (Y_j - Y_i^C) + m_{23} (Z_j - Z_i^C) \right\}$$

$$a_{26} = \frac{-f}{W} \left\{ -m_{11} (X_j - X_i^C) - m_{12} (Y_j - Y_i^C) - m_{13} (Z_j - Z_i^C) \right\}$$

$$a_{17} = -a_{11} \quad a_{27} = -a_{21}$$

$$a_{18} = -a_{12} \quad a_{28} = -a_{22}$$

$$a_{19} = -a_{13} \quad a_{29} = -a_{23}$$