Inpainting for Remotely Sensed Images With a Multichannel Nonlocal Total Variation Model

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Abstract—Filling dead pixels or removing uninteresting objects is often desired in the applications of remotely sensed images. In this paper, an effective image inpainting technology is presented to solve this task, based on multichannel nonlocal total variation. The proposed approach takes advantage of a nonlocal method, which has a superior performance in dealing with textured images and reconstructing large-scale areas. Furthermore, it makes use of the multichannel data of remotely sensed images to achieve spectral coherence for the reconstruction result. To optimize the proposed variation model, a Bregmanized-operator-splitting algorithm is employed. The proposed inpainting algorithm was tested on simulated and real images. The experimental results verify the efficacy of this algorithm.

Index Terms—Inpainting, multichannel, nonlocal total variation (NLTV), remotely sensed image.

I. INTRODUCTION

R EMOTELY sensed images provide an unparalleled data source for land-surface mapping and monitoring. However, in some situations, such as detector failure or image damage, dead pixels will exist in the remotely sensed images. Dead pixels are those pixels whose measurement does not have any correlation with the true scene that is being measured [1]. The existence of dead pixels severely degrades the quality of the imagery. There are also some situations in which we need to remove or replace certain objects from the imagery for the sake of improving its application value. For example, we can remove pedestrians on a zebra crossing to reconstruct the zebra crossing on an aerial image or remove the map lettering and labels on a raster image map to obtain the original image data. The recovery of dead pixels and the removal of selected objects from remotely sensed images can be unified into one problem, i.e., image inpainting, which has been intensively studied in the field of digital image processing [2]–[4]. The purpose of image inpainting is to reconstitute the missing or damaged portions of the image, in order to make it more legible and to restore its unity.

To solve the inpainting problem of remotely sensed images, quite a few methods have been proposed. The approaches can be grouped into three categories. The first category comprises multitemporal-complementation-based approaches. These approaches consist of selecting the best measurement among a set of measurements acquired over a limited time period. Examples of these approaches are described in the works in [7]–[11]. In [7] and [8], in order to fill the Landsat-7 scan line corrector-off (SLC-off) gaps, the authors use the data from multiple Enhanced Thematic Mapper Plus (ETM+) scenes to provide complete ground coverage. The work in [9]–[11] studies the spectrotemporal relationships between a sequence of multitemporal images for the reconstruction of areas obscured by clouds or atmospheric disturbance, by statistics, training, or classification approaches.

The second category comprises the multispectral-complementation-based approaches. Most of these approaches make use of another clear and complete band of data to recover the contaminated band of data by modeling a relationship between the contaminated band and the reference band. Examples of this class of approaches can be found in [12]–[15]. The research in [12]–[14] describes how to restore the missing data of Aqua Moderate Resolution Imaging Spectroradiometer (MODIS) band 6 by the use of other correlated bands, such as the commonly used Aqua MODIS band 7, or other image data. The work in [15] presents a haze-optimized-transformation method to detect the spatial distribution of haze and clouds in images and describes how to radiometrically adjust the visible-band imagery by the analysis of a visible-band space.

Both categories of approaches mentioned earlier need complementary information from other acquired images or spectral bands. However, in many cases, complementary images or bands cannot be acquired. Therefore, a third category of approaches is explored, which consists of filling in the missing data regions using the remaining parts in the image. The goal of the approaches in this category is to seamlessly synthesize a complete, visually plausible, and coherent image. Examples of these approaches are the recent works in [16]–[19]. The research in [16]–[18] describes how to synthesize the missing regions in remotely sensed images by propagating the geometrical structure from the remaining parts around the missing zone. In [19], the authors consider both destriping and inpainting as ill-posed inverse problems and develop a maximum a posteriori method based on a Huber–Markov model to solve these inverse problems.
problems. Furthermore, some typical examples of digital image
inpainting can be found in [2]–[6].

Since there is no need for auxiliary data, the third strategy of
approaches is more attractive. Most prior researchers in this
category have only made use of the local neighboring infor-
mation to reconstruct the missing regions in remotely sensed
images, which is far from sufficient. Moreover, few publica-
tions have considered the multiple bands of remotely sensed
images as an ensemble to do the reconstruction. To remedy
these weaknesses, this paper presents an efficient multichannel
nonlocal inpainting approach for missing data synthesis. The
proposed algorithm unites the advantages of nonlocal meth-
ods, which have a superior performance when dealing with
textured images and large areas, and local methods, which are
good at recovering geometric structures such as image edges.
Furthermore, it takes advantage of the multichannel data of
remotely sensed images to achieve spectral coherence for the
reconstruction result. That is to say, our proposed method can
achieve both the spatial and the spectral coherence. In order
to optimize the proposed multichannel nonlocal total variation
(NLTV) (MNLTV) inpainting model, a Bregmanized operator
splitting (BOS) algorithm is employed.

The rest of this paper is organized as follows. In Section II,
the basic image observation model and image inpainting model
are described. In Section III, the proposed MNLTV inpaint-
ing model is formulated. The BOS optimization method
is presented in Section IV. Section V contains the experimental
results, and Section VI is the conclusion.

II. BASIC INPAINTING MODEL

A. Image Observation Model

Assuming that we have a multispectral image with some
pixels missing, the degradation model can be written as

\[ f = Au + \varepsilon \]

(1)

where \( u = [u_1, u_2, \ldots, u_B] \) is the original true image, with
the size of \( M \times N \times B \), in which \( M \) represents the samples
of the image, \( N \) stands for the lines of the image, and \( B \)
is the number of bands. \( f = [f_1, f_2, \ldots, f_B] \) is the observed
degradation image, which is also of size \( M \times N \times B \). \( A \) is a
diagonal matrix with diagonal elements consisting of 0 and 1,
with 0 representing the missing data. \( \varepsilon \) is additive noise with
the same size as \( u \) and \( f \). Our objective is to find the unknown
target image \( u \) from the observed image \( f \).

B. Image Inpainting Model

The multispectral image inpainting process is essentially an
ill-posed inverse problem, which is similar to many other image
processing problems, such as image denoising [20], destrip-
ing [19], superresolution reconstruction [21], [22], and others.
The work in [23] and [24] provided approaches that achieve
superresolution reconstruction and inpainting simultaneously.
It is standard to use a regularization technique to make these
inverse problems well posed. Regularization methods assume
some prior information about the unknown image \( u \), such as
smoothness, sparsity [25], manifold [26], or small TV [31].

Based on a regularization technique, the inpainting problem
for a multispectral image can be represented by the following
model:

\[ \hat{u} = \arg \min_u J(u) \quad \text{s.t. } Au = f \]

(2)

where \( J(u) \) is the regularization item giving a prior model of the
target image. The corresponding constrained problem for a
noisy case is then written as

\[ \hat{u} = \arg \min_u J(u) \quad \text{s.t. } \|Au - f\|^2 \leq \sigma \]

(3)

where \( \sigma \) is the standard deviation of the noise \( \varepsilon \).

III. MNLTV METHOD

Almost all the regularization methods mentioned earlier such
as sparsity, manifold, and TV regularization belong to local
methods which recover a pixel using only the local neighboring
information; it is insufficient. In recent years, nonlocal methods
for image denoising and inpainting have gained considerable
attention. This is partly due to their superior performance in
dealing with textured images. Local methods, on the other hand,
have proved to be very effective for the recovery of geometric
structure such as image edges. The synthesis of both types of
methods is an important research area. Variation analysis is an
appropriate tool for the unification of local and nonlocal meth-
ods. In recent research, single-channel nonlocal regularization
has been developed for digital image processing [27], [28],
and it has proved to be very effective. In order to take advantage
of the multichannel data of remotely sensed images, an MNLTV
inpainting model is presented in this paper.

A. Nonlocal Filter

The nonlocal methods in image processing are generalized
from the Yaroslavsky filter and patch-based methods. The idea
is to restore an unknown pixel using other similar pixels. The
resemblance is regarded in terms of a patch centered at each
pixel, not just the intensity of the pixel itself. In order to restore
a pixel, the nonlocal methods average the other pixels with
structures (patches) similar to that of the current one. This idea
was generalized to a famous neighborhood denoising filter, the
nonlocal means (NL-means) by Buades et al. in [29]. More pre-
cisely, given a reference image \( f \), \( \Omega \) is its pixel domain. We de-
define the NL-means solution \( \text{NLM}_u \) of the image \( u \) at point \( x \) as

\[ \text{NLM}_u(x) := \frac{1}{C(x)} \sum_{y \in \Omega} w_f(x, y) f(y) \]

(4)

where

\[ w_f(x, y) = \exp \left\{ - \frac{\left( G_{\alpha} f(x + \cdot) - f(y + \cdot) \right)^2}{h^2} (0) \right\} \]

(5)

\[ C(x) = \sum_{y \in \Omega} w_f(x, y) \]

(6)
where $G_a$ is the Gaussian kernel with standard deviation $a$, $C(x)$ is the normalizing factor, $h$ is a filtering parameter, and $f(x+\cdot)$ can be known as a patch centered at a point $x$. The patch $f(x+\cdot)$ of size $m \times m$ ($m$ is chosen as an odd number) is given as (7), and the size of the patch is according to the noise intensity

$$f(x+\cdot) = f(x+t), \quad t = \left[ -\frac{m-1}{2}, \ldots, \frac{m+1}{2} \right]. \quad (7)$$

We recall that

$$\left( G_a \left| f(x+\cdot) - f(y+\cdot) \right|^2 \right)(0) = \sum G_a(t) \left| f(x+t) - f(y+t) \right|^2. \quad (8)$$

Following (5), we can compute a weight function $w(x,y)$ between two points $x$ and $y$ by using the difference of patches around each point. This choice of weight is very efficient in reducing noise while preserving the textures and contrast of natural images. It is to be noted that the missing points are not needed to be excluded from the convolution summation since this operation does not affect the final solution of the proposed variation model.

### B. Nonlocal Operators

In order to formulate the NL-means filter in a variational framework, Gilboa and Osher [27] defined variational framework-based nonlocal operators. In the following, we give the definitions of the nonlocal functions introduced in [27]. Let $\Omega \subset \mathbb{R}^2$, $x, y \in \Omega$, $u(x)$ be a real function $u : \Omega \rightarrow \mathbb{R}$, and $w(x,y)$ be a weight function. Furthermore, $w(x,y)$ is assumed to be nonnegative and symmetric.

Nonlocal gradient $\nabla_w u(x) : \Omega \rightarrow \Omega \times \Omega$ is defined as the vector of all partial derivatives $\nabla_w u(x,\cdot)$ at $x$ such that

$$(\nabla_w u)(x,y) := (u(y) - u(x)) \sqrt{w(x,y)}, \quad \forall y \in \Omega. \quad (9)$$

We denote vectors as $\overrightarrow{p} = p(x,y) \in \Omega \times \Omega$, and the nonlocal divergence $(\text{div}_w \overrightarrow{p})(x) : \Omega \times \Omega \rightarrow \Omega$ is defined as the adjoint of the nonlocal gradient

$$(\text{div}_w \overrightarrow{p})(x) := \sum_{y \in \Omega} (p(x,y) - p(y,x)) \sqrt{w(x,y)}. \quad (10)$$

The nonlocal $H^1$ norm and the NLTV are defined as follows:

$$J_{NLH^1}^w(u) := \sum_{x \in \Omega} |\nabla_w u(x)|^2 = \sum_{x \in \Omega} \sum_{y \in \Omega} \frac{(u(x) - u(y))^2}{\sqrt{w(x,y)}}. \quad (11)$$

$$J_{NLTV}^w(u) := \sum_{x \in \Omega} |\nabla_w u(x)| = \sum_{x \in \Omega} \sqrt{\sum_{y \in \Omega} (u(x) - u(y))^2 w(x,y)}. \quad (12)$$

In this paper, we are interested in NLTV because, analogous to classical TV, the $L^1$ norm is generally more efficient than the $L^2$ norm for sparse reconstruction.

### C. MNLTV

For multichannel images, Blomgren and Chan [30] presented a multichannel TV (MTV) regularization by coupling the channels

$$J_{MTV}(u) := \sum_{x \in \mathbb{M} \times \mathbb{N}} \sqrt{\sum_{j=1}^{B} |\nabla u_j(x)|^2} \quad (13)$$

$$\nabla u_j(x) = \sqrt{(u_j(x) - u_j(rx))^2 + (u_j(x) - u_j(bx))^2} \quad (14)$$

where $rx$ and $bx$ represent the nearest neighbor to the right and below the pixel, respectively. The work of Yuan et al. [31] demonstrated that this MTV model has a powerful spectrally adaptive ability in remotely sensed image processing.

Inspired by the previously mentioned work, we extend NLTV to MNLTV and propose an MNLTV regularization for multi-spectral images

$$J_{MNLTV}^w(u) := \sum_{x \in \mathbb{M} \times \mathbb{N}} \sum_{j=1}^{B} \sqrt{\sum_{y \in \mathbb{M} \times \mathbb{N}} (u_j(x) - u_j(y))^2 w(x,y)}. \quad (15)$$

### D. MNLTV Inpainting Models

According to the inpainting models mentioned in Section II, the corresponding MNLTV inpainting model is

$$\hat{u} = \arg\min_u J_{MNLTV}^w(u), \quad \text{s.t.} \quad Au = f. \quad (16)$$

In a noisy case, the model is then written as

$$\hat{u} = \arg\min_u J_{MNLTV}^w(u), \quad \text{s.t.} \quad \|Au - f\|^2 \leq \sigma. \quad (17)$$

### IV. Optimization

The Bregman methods for image processing introduced in [32] have been demonstrated to be efficient optimization methods for solving sparse reconstruction, such as the $l^1$ norm and TV. Recently, based on a Bregman method, a well-performing optimization algorithm called BOS [33] was developed to provide a general algorithm framework for equality-constrained convex optimization. In this paper, the BOS algorithm is extended and used to optimize the MNLTV inpainting model in (16) and (17). The basic idea of this optimization algorithm can be stated as follows.

First, the constraint problems in (16) and (17) are enforced with the Bregman iteration process

$$\begin{cases}
  u^{k+1} = \arg\min_u \left( \mu J_{MNLTV}^w(u) + \frac{1}{2} \|Au - f^k\|^2 \right) \\
  f^{k+1} = f + \hat{u}^{k+1} - Au^{k+1}.
\end{cases} \quad (18)$$

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where $\mu$ is a positive parameter $\mu > 0$; it is the regularization term scale. The first subproblem can sometimes be difficult and slow to solve directly since it involves the inverse of the operator $A$ and the convex function $J^w_{\text{MNLTV}}$ in (16). The forward–backward operator splitting technique is used to solve the unconstrained subproblem in (18) as follows: For $i \geq 0$, $u^{k+1,i} = u^k$

$$
\begin{align*}
\{ u^{k+1,i+1}, v^{k+1,i+1} \} &= \arg \min_u \left\{ \mu J^w_{\text{MNLTV}}(u) + \frac{1}{2\lambda} \| u - v^{k+1,i+1} \|^2 \right\} \\
&= \arg \min_u \left\{ \mu J^w_{\text{MNLTV}}(u) + \frac{1}{2\lambda} \| u - v^{k+1,i+1} \|^2 \right\}
\end{align*}
$$

(19)

where $\delta$ is a positive parameter such that $0 < \delta < (2/\|A\|)$. We can see that the efficiency of the BOS algorithm depends on solvers for the $u$ subproblem in (19). Here, we extend the split Bregman method proposed by Goldstein and Osher in [34] to the multichannel nonlocal case. It has proved to be a fast and efficient algorithm to minimize the MNLTV function in the subproblem

$$
\hat{u} = \arg \min_u \left( \mu \delta J^w_{\text{MNLTV}}(u) + \frac{1}{2} \| u - v \|^2 \right).
$$

(20)

The idea is to reformulate the problem as

$$
\hat{u} = \min_u \left( \mu \delta \sum_{x \in M \times N} \sqrt{\frac{B}{\sum_{j=1}^B d_j(x)^2}} + \frac{1}{2} \| u - v \|^2 \right)
$$

subject to $d_j = \nabla w u_j$.

(21)

By enforcing the constraint problem with a Bregman iteration process, the extended MNLTV split Bregman algorithm using the MNLTV norm is given by

$$
\begin{align*}
(u^{k+1}, v^{k+1}) &= \min_u \left( \mu \delta \sum_{x \in M \times N} \sqrt{\sum_{j=1}^B d_j(x)^2} \\
&\quad+ \frac{1}{2} \| u - v \|^2 + \frac{\lambda}{2} \| d - \nabla w u - b_k \|^2 \right) \\
&= \arg \min_u \left( \mu \delta \sum_{x \in M \times N} \sqrt{\sum_{j=1}^B d_j(x)^2} \right)
\end{align*}
$$

(22)

where $\lambda$ is the scale of penalty term $\| d - \nabla w u - b_k \|^2$; it is usually inversely proportional to the value of $\mu \cdot \delta$. The solution of (22) is obtained by performing alternately with the following two subproblems:

$$
\begin{align*}
u^{k+1} &= \min_u \left( \frac{1}{2} \| u - v \|^2 + \frac{\lambda}{2} \| d - \nabla w u - b_k \|^2 \right) \\
v^{k+1} &= \min_d \left( \mu \delta \sum_{x \in M \times N} \sqrt{\sum_{j=1}^B d_j(x)^2} \right)
\end{align*}
$$

(23)

The subproblem for $u^{k+1}$ consists of solving the linear system

$$
(u^{k+1} - v) - \lambda \text{div} w (\nabla u^{k+1} + b_k - d^k) = 0.
$$

(24)

As the linear function in (24) is strictly diagonal, we can solve $u^{k+1}$ by a Gauss–Seidel algorithm.

The $d^{k+1}$ subproblem equation in (23) can be solved using a shrinkage operator as follows:

$$
d^{k+1} = \text{shrink} \left( \sum_{j=1}^B (\nabla w u_{j}^{k+1} + b_k^2 - \frac{\mu \delta}{\lambda}) \right)
$$

(25)

where

$$
\text{shrink}(x, \tau) = \frac{x}{|x|} \max(|x| - \tau, 0).
$$

(26)

The weights computed from the initial image are not generally sufficient to give a good estimation. Therefore, it is necessary to adapt the weight used in nonlocal regularization, according to (5), during the iteration.

The optimization procedure of the MNLTV inpainting model is described in Algorithm 1.

**Algorithm 1. MNLTV Inpainting Algorithm.**

Initialization: $u^0 = v^0 = 0$, $\Delta u^0 = f$, $n\text{Outer}$, $n\text{Inner}$. While $k < n\text{Outer}$ and $\| \Delta u^k - f \| > \text{tolerance}$ do

- for $i = 0$ to $n\text{Inner}$ do
  - $u^{k+1,i} = u^k$;
  - update $v^{k+1,i+1} = u^{k+1,i} - \delta A(u^{k+1,i} - f^k)$;
  - solve $u^{k+1,i+1}$ by the split Bregman algorithm:

$$
u^{k+1,i+1} = \arg \min_u \left( \mu \delta J^w_{\text{MNLTV}}(u) + \frac{1}{2} \| u - v^{k+1,i+1} \|^2 \right)
$$

end

- update the nonlocal weight according to $u^{k+1}$: $w(u^{k+1})$ by (5);
- update $f^{k+1} = f^k + \Delta u^k$;
End

The convergence of the BOS optimization algorithm has been proved theoretically in [33]. Moreover, we experimentally verified that it has a very fast convergence rate. Then, the computational complexity of the proposed algorithm is provided. We assume that the number of the bands of the input image is $L$, and the total number of the pixels in each single band is $N$. Then, in Algorithm 1, the computational complexity for updating $v$ is $O(NL)$. In the step of solving $u$ by the split Bregman algorithm, it is resolved into two subproblems as in (23), and the $u$ subproblem is just a fast Gauss–Seidel algorithm, with a linear computational complexity of $O(N^2 L)$, while the $d$ subproblem is an efficient soft threshold/shrinkage operator, with a linear computational complexity of $O(N^2 L^2)$, and then, in the update of nonlocal weight $w$, the computational complexity is $O(N^3 L)$. Finally, the computational complexity for updating $f$ is $O(NL)$. Taking all the aforementioned parts into account, the total computational complexity for Algorithm 1 is $O(N^2 L^2)$. 

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Fig. 1. Experimental results for the recovery of noise and vertical dead lines. (a) Original image. (b) Simulated image contaminated by noise and dead lines. Recovered images using the following: (c) Proposed algorithm, (d) NLTV inpainting algorithm, (e) MTV inpainting model, (f) NL-means inpainting method, (g) MCA algorithm, and (h) Criminisi method.

V. EXPERIMENTS

A. Experiments on Simulated Data

In the simulation experiments, we conduct three experiments to test and verify the efficacy of our proposed MNLTV inpainting algorithm. Results of the three experiments are shown in Figs. 1–4. The peak signal-to-noise ratio (PSNR) index is used to give a quantitative assessment of the results of the simulated experiments from the gray-level fidelity aspect. The structural similarity (SSIM) index [35] is used to give a quantitative assessment of the results from the structure-level fidelity aspect. Recently, some image quality assessment indices based on human vision system have been proposed, such as metric $Q$ [36] and the multiscale geometric analysis-based indices [37], which provide a quantitative measure of image content (i.e., sharpness and contrast in visually salient geometric features such as edges). In this paper, we use the metric $Q$ index to give a quantitative assessment of the inpainting result from a human vision aspect

$$\text{PSNR} = 10 \log_{10} \left( \frac{255^2 \cdot MN}{\| \hat{u} - \hat{u} \|^2} \right)$$  \hspace{1cm} (27)$$

$$\text{SSIM} = \frac{(2\mu_u \mu_{\hat{u}} + C_1)(2\sigma_{u\hat{u}} + C_2)}{(\mu_u^2 + \mu_{\hat{u}}^2 + C_1)(\sigma_u^2 + \sigma_{\hat{u}}^2 + C_2)}$$  \hspace{1cm} (28)$$

$$Q = s_1 \frac{s_1 - s_2}{s_1 + s_2}$$  \hspace{1cm} (29)$$

where $MN$ is the total number of pixels in the multichannel image, $\hat{u}$ and $u$ represent the recovered image and the original clear image, and $\mu_u$ and $\mu_{\hat{u}}$ represent the average gray values of the original clear image and the recovered image, respectively. $\sigma_u$ and $\sigma_{\hat{u}}$ represent the variances of the original clear image and the recovered image, respectively. $\sigma_{u\hat{u}}$ represents the covariance between the original clear image and the recovered image. $C_1$ and $C_2$ are two constants, $s_1$ and $s_2$ are two singular values of the gradient matrix of the recovered image.

The first simulated data test of the proposed algorithm is for the recovery of vertical dead lines in a QuickBird image with the resolution of 0.6 m. We use three bands of the red, green, and blue to undertake this test. Fig. 1(a) shows the original subimage. In this experiment, the simulated image is contaminated by dead lines of 7-pixel widths. Moreover, we simulate the additive noise by adding different variance
zero-mean Gaussian noise in different bands. This is shown in Fig. 1(b). To make a comparative analysis, the proposed MNLTV inpainting algorithm is compared with the NLTV inpainting algorithm [27], the MTV inpainting algorithm [31], the NL-means inpainting method [29], the morphological component analysis (MCA) method [5], and the Criminisi method [6]. The recovery results of each method are shown in Fig. 1(c)–(h).

From Fig. 1 and its zoomed detailed regions in Fig. 2, it can be seen that the proposed MNLTV algorithm gives better denoising and inpainting results, compared to the results of the other methods. In the MNLTV result, on the one hand, the noise is suppressed more thoroughly, and on the other hand, the recovery of dead lines is spatially continuous, with more convincing visual quality. The result of the NLTV algorithm is not as sharp as the result of our proposed algorithm, and some noise still remains in the smooth regions because the noise intensity difference between different bands is not taken into consideration and an equal denoising strength is used in all bands. For the MTV algorithm, the result suffers from a “staircasing” effect; moreover, it is incapable of performing narrow and long
region connection. In the result of the NL-means inpainting method, the narrow and long regions also cannot be connected completely, and the denoising result is oversmoothed. The result using the MCA algorithm has a strong ripple effect and spectral distortion, and the recovery of the dead lines shows severe artifacts, particularly in the homogeneous region. From the result of the Criminisi method, we can see that it is incapable of denoising at all: The noise in the resulting image has the same intensity as the contaminated image, and a lot of spurious detailed information appears in the recovery result of the dead region due to its block filling process.

It is to be noted that all the test image data in our paper are red, green, and blue true color composite of the original acquired data.

Next, the proposed algorithm is tested with another type of inpainting problem, in which dead pixels are randomly distributed. Fig. 3(a) shows an original GeoEye-1 image with the resolution of 1.65 m. In this experiment, the simulated image is contaminated by 50% random dead pixels in all bands, as shown in Fig. 3(b). The proposed MNLTV algorithm is compared with the NLTV algorithm, MTV model, and MCA algorithm. The other two comparative methods used in the first experiment, the NL-means inpainting method and the Criminisi method, fail to work in this experiment because the missing data are distributed in almost every patch of the image and occupy a large-scale area. The ideas of patch matching and patch filling, which the other two methods are based on, cannot work in this situation. The recovery result of each method is shown in Fig. 3.

From Fig. 3, it can be seen that our proposed algorithm achieves a nice inpainting result. The image quality is very good, with most of the detailed information recovered well, and the ground objects can be clearly distinguished. Although the result of the NLTV algorithm is quite similar to that of the proposed MNLTV algorithm from a visual perspective, the quantitative measure of our proposed algorithm is improved, as shown in Table I. In the result of the MTV algorithm, we can see that it shows some blurring, and some edge information is not recovered completely. For the MCA algorithm, the ripple effect still exists.

The third simulation experiment involves the removal of unwanted map lettering on remotely sensed images. Remote sensing mapping, as a branch of cartography, provides a significant application for remotely sensed images and makes maps more intuitive and richer in content. This kind of map, which is known as a photographic map or image map, is made up of the appropriate remotely sensed image and several auxiliary feature notes on the image. Once the map is printed out or rasterized, the feature notes cannot be separated from the image, i.e., we cannot regain the original image. However, we sometimes need to get the original remotely sensed image from a map for other uses, particularly when the image data are rare or valuable. In such a case, the map lettering needs to be removed, and the original image needs to be reconstructed. The proposed MNLTV method can be used to efficiently undertake this task.

In this experiment, figure notes were manually created on an aerial image in order to simulate a photographic map, as shown in Fig. 4(b). The original aerial image with a 0.2-m resolution is shown in Fig. 4(a). Fig. 4(c) shows the result of our proposed method. The results of the other comparative methods are shown in Fig. 4(d)–(h).

Fig. 4(c) shows that our proposed method is capable of removing the map lettering and recovering a clear image that is much closer to the actual image. Although it may not be exactly precise, the visual quality is convincing enough. In the results
TABLE I

<table>
<thead>
<tr>
<th>Contaminated Image</th>
<th>PSNR</th>
<th>SSIM</th>
<th>Metric Q Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSNR</td>
<td>15.68</td>
<td>0.7178</td>
<td>69.66</td>
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<tr>
<td>Fig. 1</td>
<td></td>
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<td>62.79</td>
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<td>Fig. 4</td>
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Fig. 5. (a)–(h) Detailed regions cropped from Fig. 4(a)–(h).

of the NLTV inpainting algorithm and the NL-means inpainting algorithm, it appears that there is spectral distortion in some inpainted regions. The results of the other approaches all show the same flaws as the results of the foregoing experiments. For the convenience of visual judgment, a series of detailed regions cropped from Fig. 4(a)–(h) is shown in Fig. 5(a)–(h).

The effectiveness of the proposed MNLTV inpainting method can also be illustrated by the quantitative assessment. The PSNR, SSIM, and metric Q values of Figs. 1, 3, and 4 are shown in Table I to give an overall quantitative assessment of the results. It can be seen from Table I that, for the results in every simulation experiment, the PSNR values using the MNLTV model are higher than those of the other methods. All three groups of simulated data experimental results indicate that the proposed MNLTV method can provide a better and more robust inpainting result.

In nonlocal methods, the filtering parameter $h$ in (5) can exert considerable influence on the inpainted image. Here, we test the sensitivity of this parameter. For each of the three sets of simulated experimental data, we select a series of different $h$'s to implement the inpainting methods of NLTV and the proposed MNLTV. The relationship of the acquired PSNR value and $h$ is shown in Fig. 6. From Fig. 6(a)–(c), it can be seen that the PSNR values using the MNLTV method fluctuate less than those using the NLTV method with the change in $h$. On two sides of the peak point, the PSNR values using the MNLTV method descend much more slowly, while the PSNR values using the NLTV method show a dramatic decline. This
phenomenon can be explained as follows. From (5), we can see that \( h \) indicates the exponential decay rate that is also the decay rate of weight \( W \). The decay of \( W \) has a significant effect on the nonlocal gradient \( \nabla w_j(x) \) from (9) within each band. However, from (15), we can deduce the multichannel nonlocal gradient magnitude at point \( x \):

\[
|\nabla_{mw} u(x)| = \sqrt{\sum_{j=1}^{B} |\nabla w_j(x)|^2}.
\]  

(30)

It can be seen that, for the MNLTV model, the nonlocal gradient magnitude of every point is determined by the combination of multichannel data, so it will be more steady and robust than just using a single band of data, as in the NLTV model.

### B. Real Data Experiments

The SLC of the Landsat-7 ETM+ sensor failed in 2003, resulting in about 22% of the pixels per scene not being scanned. The SLC failure has seriously limited the scientific application of ETM+ data. Our first real data experiment involves filling the gaps in a Landsat-7 ETM+ SLC-off image. The test image is shown in Fig. 7(a), with a size of 200 \( \times \) 200 pixels and a 4–5-pixel width of dead lines, and the resolution is 30 m. The result of our proposed MNLTV method is shown in Fig. 7(b). The other comparative results are shown in Fig. 7(c)–(f).

From Fig. 7 we can see that the image using our proposed method has a better filling result than that of the other existing methods: It appears more spatially continuous, without stripes, and the edge information and the detailed information are well filled. However, in the results of the MTV and NL-means inpainting method, the narrow and long regions cannot be connected. The result using the MCA algorithm shows some artifacts in the homogeneous vegetation region. There are still visible stripes left in the result of the exemplar-based method. For the convenience of visual judgment, a series of detailed regions cropped from Fig. 7(a)–(h) are shown in Fig. 8(a)–(h).

In the second real data experiment, the proposed MNLTV algorithm is tested with the reconstruction of a zebra crossing.
Fig. 8. (a)–(f) Detailed regions cropped from Fig. 7(a)–(f).

Fig. 9. Experimental results for the reconstruction of a zebra crossing. (a) Original image. (b) Zebra crossing contaminated with pedestrians. (c) Mask of the area to be reconstructed. Reconstructed zebra crossings using the following: (d) Proposed algorithm, (e) MTV inpainting model, (f) NL-means inpainting method, (g) MCA algorithm, (h) Criminisi method, and (i) Reconstructed image.
on an aerial image. With high-resolution images providing an important data source for terrain observation, how to extract the required geoinformation rapidly and accurately has become an important research field. Traffic signs are important land objects in high-resolution images, such as the zebra crossing, which marks a safe area for people to pass across roads. However, in many cases, zebra crossings in high-resolution images are contaminated with other features, such as pedestrians, as shown in Fig. 9(a), which seriously interfere with the recognition and extraction of the zebra crossing. Therefore, it is valuable and necessary to reconstruct zebra crossings in images, not only for enriching the information of a map but also for prompting warnings in vehicle navigation, which helps to reduce the occurrence of traffic accidents.

This experiment is performed on an aerial image with a resolution of 0.1 m, as shown in Fig. 9(a). Our goal is to remove the pedestrians on the zebra crossing and reconstruct a complete zebra crossing. Fig. 9(c) shows the portion of the pedestrian-contaminated region that should be reconstructed (the black region). The region to be reconstructed is manually segmented by the user, for the moment. A further investigation of the pedestrian-contaminated area segmentation could help to automate the entire reconstruction process. The reconstructed results are shown in Fig. 9(d)–(h). It can be seen that we finally get an image with a complete zebra crossing in it, as shown in Fig. 9(i). The zebra crossing recovery results shown in Fig. 9 indicate that the proposed MNLTV algorithm is able to perform repetitive texture reconstruction well. The missing regions reconstructed using our proposed method achieve both the spatial and the spectral consistency with the surrounding texture.

In the third real data experiment, the proposed inpainting method is tested on an image map. The aim is to reconstruct image data without map lettering. The map that we used as the test data is an area of Washington, DC. Fig. 10(a) shows a part of this map, with a size of 350 × 300 pixels. The reconstructed result of our proposed MNLTV method is shown in Fig. 10(b). The other four comparative results are shown in Fig. 10(c)–(f). A series of detailed regions cropped from Fig. 10(a)–(f) is shown in Fig. 11(a)–(f). From the results, it can be seen that our proposed MNLTV method is able to remove the map lettering and reconstruct different ground objects well; however, it still produces blurring to a certain extent in the regions where the ground features are complex. Another competitive result for this image is that of the Criminisi method.

The metric $Q$ values of Figs. 7, 9, and 10 are shown in Table II to give an overall quantitative assessment of the three sets of real data experimental results.

**VI. Conclusion**

In this paper, we present an MNLTV inpainting model to deal with the remotely sensed image reconstruction problem. The proposed algorithm was applied to Landsat-7 ETM+ SLC-off image gap filling, zebra crossing reconstruction from an aerial image, and map lettering removal from a photographic map. All the simulated data experiments and real data experiments indicate that the reconstruction results using our proposed
algorithm are very effective, no matter whether the image is noisy or not, how the missing data are distributed, or whether the reconstructed region is homogeneous, edge or texture. Furthermore, through an analysis of the filtering parameter, it was demonstrated that the proposed MNLTV model is more robust than the NLTV model. Nevertheless, there may still be room for improvement of our proposed method. In the proposed MNLTV model, the nonlocal weighting function calculation [see (5)] is restricted to within a single band, and our future work will focus on extending the weight calculation from a single band to multiple bands. Moreover, some regularization parameters such as $\mu$ in (18) and $\lambda$ in (22) are set fixedly by manual selection, and we will try to make them to be automatic and self-adapting in the future work.

**REFERENCES**


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